 FYUESE – Math-IRC-1

2022-26

Full Marks : 100

Time : 3 Hours

Candidates are required to give their answer in their own words as far as practicable. Their figures in the margin indicate full marks.

Answer from **both** the Groups as directed.

Group – A

(Compulsory)

1. Answer the following questions : 1×10=10

(a) State Euler's Theorem.

(b) State Leibnitz's Theorem.

(c) Define Indexed sets.

(d) Define union for an indexed family of sets.

(e) Define Translation of axes.

(f) Define Rotation of axes.

(g) Define scalar point function.

(h) Define vector function.

(i) State De-Moivre's Theorem.

(j) Find the value of $(\text{Cos}\theta - i\sin\theta)^{-7}$.

2. If u=log $(x^3+y^3+z^3-3xyz)$ 5

Prove that : $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$

3. Prove that : 5

$$(a+ib)^{\frac{m}{n}}+(a-ib)^{\frac{m}{n}}=2\left(a^2+b^2\right)^{\frac{m}{2n}}\cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$$

Group – B

Answer any **four** questions of the following :

4. (a) If y=a cos(log x)+b sin (log x) 10

Prove that :

$$x^2y_{n+2}+(2n+1)xy_{n+1}+\left(n^2+1\right)y_n=0$$

(b) If $u=e^{xyz}$ 10

Prove that:

$$\frac{\partial^3 u}{\partial x\,\partial y\,\partial z}=\left(1+3xyz+x^2y^2z^2\right)e^{xyz}$$

5. (a) Evaluate : 10

$$\int\frac{(x-2)dx}{(x-1)(x-5)}$$

(b) Evaluate : 10

$$\int \frac{dx}{\sqrt{(x-a)(x-b)}}$$

6. (a) If $\vec{a}$ is a unit vector, then 10

Prove that :

$$\left|\vec{a} \times \frac{d\vec{a}}{dt}\right| = \left|\frac{d\vec{a}}{dt}\right|$$

(b) If $\vec{r} = \cos \omega t \vec{a} + \sin \omega t \vec{b}$, 10

Prove that :

(i) $\vec{r} \times \frac{d\vec{r}}{dt} = \omega \vec{a} \times \vec{b}$

(ii) $\frac{d^2\vec{r}}{dt^2} + \omega^2 \vec{r} = 0$

Where $\vec{a}$ & $\vec{b}$ are constant veotors.

7. (a) Transform to axes inclined at 30° to the original axes the equation : 10

$x^2 + 2\sqrt{3}\, xy - y^2 = 2a^2$

(b) Transform the equation : 10

$x^2 + y^2 - 4x + 8y - 17 = 0$ to Parallel axes through the point (2,–4).

8. (a) State and prove De Morgan's laws in general form. 10

(b) If $\{Ai\}_{i\in I}$ be an indexed family of sets than for any set B, Prove that :

(i) $B \cup \left(\bigcap_{i\in I} A_i \right) = \bigcap_{i\in I} (B \cup A_i)$ 5

(ii) $B \cap \left(\bigcup_{i\in I} A_i \right) = \bigcup_{i\in I} (B \cap A_i)$ 5

9. (a) If $x_r = \cos\frac{\pi}{z^r} + i\sin\frac{\pi}{z^r}$ 10

Prove that $x_1 . x_2 . x_3 \ldots\ldots \text{ to } \infty = -1$

(b) Find the equation whose roots are the nth powers of the roots of the equation $x^2 - 2x\cos\theta + 1 = 0$ 10

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