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Yogoda Satsanga Mahavidyalaya

Jagannathpur, Dhurwa, Ranchi-834004

www.ysmranchi.net

ysmranchi4@gmail.com

Course :Computer System Architecture

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Lesson :Canonical Form

By :Goutam Sanyal

Canonical Form

Canonical and Standard Forms

- We need to consider formal techniques for the simplification of Boolean functions.
  - Identical functions will have exactly the same canonical form.
  - Minterms and Maxterms
  - Sum-of-Minterms and Product-of- Maxterms
  - Product and Sum terms
  - Sum-of-Products (SOP) and Product-of-Sums (POS)

Definitions

- *Literal:* A variable or its complement
- *Product term:* literals connected by •
- *Sum term:* literals connected by +
- *Minterm:* a product term in which all the variables appear exactly once, either complemented or uncomplemented
- *Maxterm:* a sum term in which all the variables appear exactly once, either complemented or uncomplemented

Minterm

- Represents exactly one combination in the truth table.
- Denoted by m_j, where j is the decimal equivalent of the minterm's corresponding binary combination (b_j).
- A variable in m_j is complemented if its value in b_j is 0, otherwise is uncomplemented.
- Example: Assume 3 variables (A,B,C), and j=3. Then, b_j = 011 and its corresponding minterm is denoted by m_j = A'BC

Maxterm

- Represents exactly one combination in the truth table.
- Denoted by M_j, where j is the decimal equivalent of the maxterm's corresponding binary combination (b_j).
- A variable in M_j is complemented if its value in b_j is 1, otherwise is uncomplemented.
- Example: Assume 3 variables (A,B,C), and j=3. Then, b_j = 011 and its corresponding maxterm is denoted by M_j = A+B'+C'

Truth Table notation for Minterms and Maxterms

- Minterms and Maxterms are easy to denote using a truth table.
- Example:
  Assume 3 variables x,y,z
  (order is fixed)

x	y	z		Minterm	Maxterm
0	0	0		$x'y'z' = m_0$	$x+y+z = M_0$
0	0	1		$x'y'z = m_1$	$x+y+z' = M_1$
0	1	0		$x'yz' = m_2$	$x+y'+z = M_2$
0	1	1		$x'yz = m_3$	$x+y'+z' = M_3$
1	0	0		$xy'z' = m_4$	$x'+y+z = M_4$
1	0	1		$xy'z = m_5$	$x'+y+z' = M_5$
1	1	0		$xyz' = m_6$	$x'+y'+z = M_6$
1	1	1		$xyz = m_7$	$x'+y'+z' = M_7$

Canonical Forms (Unique)

- Any Boolean function F( ) can be expressed as a *unique* **sum** of **min**terms and a unique **product** of **max**terms (under a fixed variable ordering).
- In other words, every function F() has two canonical forms:
  - Canonical Sum-Of-Products (sum of minterms)
  - Canonical Product-Of-Sums (product of maxterms)

Example

- Truth table for $f_1(a,b,c)$ at right
- The canonical sum-of-products form for f_1 is
  $f_1(a,b,c) = m_1 + m_2 + m_4 + m_6$
  $= a'b'c + a'bc' + ab'c' + abc'$
- The canonical product-of-sums form for f_1 is
  $f_1(a,b,c) = M_0 \bullet M_3 \bullet M_5 \bullet M_7$
  $= (a+b+c)\bullet(a+b'+c')\bullet$
  $(a'+b+c')\bullet(a'+b'+c')$.
- Observe that: $m_j = M_j'$

a	b	c		f_1
0	0	0		0
0	0	1		1
0	1	0		1
0	1	1		0
1	0	0		1
1	0	1		0
1	1	0		1
1	1	1		0

Shorthand: ∑ and ∏

- $f_1(a,b,c) = \sum m(1,2,4,6)$, where ∑ indicates that this is a sum-of-products form, and m(1,2,4,6) indicates that the minterms to be included are m_1, m_2, m_4, and m_6.
- $f_1(a,b,c) = \prod M(0,3,5,7)$, where ∏ indicates that this is a product-of-sums form, and M(0,3,5,7) indicates that the maxterms to be included are M_0, M_3, M_5, and M_7.
- Since $m_j = M_j'$ for any *j*,
  $\sum m(1,2,4,6) = \prod M(0,3,5,7) = f_1(a,b,c)$

Conversion Between Canonical Forms

- Replace ∑ with ∏ (or *vice versa*) and replace those *j'*s that appeared in the original form with those that do not.
- Example:

$f_1(a,b,c)$ = a'b'c + a'bc' + ab'c' + abc'

$= m_1 + m_2 + m_4 + m_6$

$= \sum(1,2,4,6)$

$= \prod(0,3,5,7)$

= (a+b+c)•(a+b'+c')•(a'+b+c')•(a'+b'+c')

Standard Forms (NOT Unique)

- Standard forms are "*like*" canonical forms, except that not all variables need appear in the individual product (SOP) or sum (POS) terms.
- Example:
  $f_1(a,b,c) = a'b'c + bc' + ac'$
  is a *standard* sum-of-products form
- $f_1(a,b,c) = (a+b+c)\bullet(b'+c')\bullet(a'+c')$
  is a *standard* product-of-sums form.

Conversion of SOP from standard to canonical form

- Expand *non-canonical* terms by inserting equivalent of 1 in each missing variable x:
  $(x + x') = 1$
- Remove duplicate minterms
- $f_1(a,b,c) = a'b'c + bc' + ac'$
  $= a'b'c + (a+a')bc' + a(b+b')c'$
  $= a'b'c + abc' + a'bc' + abc' + ab'c'$
  $= a'b'c + abc' + a'bc + ab'c'$

Conversion of POS from standard to canonical form

- Expand noncanonical terms by adding 0 in terms of missing variables (*e.g.*, xx' = 0) and using the distributive law
- Remove duplicate maxterms
- $f_1(a,b,c)$ = (a+b+c)•(b'+c')•(a'+c')
  = (a+b+c)•(aa'+b'+c')•(a'+bb'+c')
  = (a+b+c)•(a+b'+c')•(a'+b'+c')•
  (a'+b+c')•(a'+b'+c')
  = (a+b+c)•(a+b'+c')•(a'+b'+c')•(a'+b+c')

Canonical SOP Form Using Truth Table

Designation	a	b	c	a’b’c + bc’ + ac’
0	0	0	0	1.1.0+0.1+0.1=0
1	0	0	1	1.1.1+0.0+0.0=1
2	0	1	0	0.0.0+1.1+0.1=1
3	0	1	1	1.0.1+1.0+0.0=0
4	1	0	0	0.1.0+0.1+1.1=1
5	1	0	1	0.1.0+0.0+1.0=0
6	1	1	0	0.0.1+1.1+1.1=1
7	1	1	1	0.0.1+1.0+1.0=0

F(a,b,c)=∑(1,2,4,6)= a’b’c + abc’ + a’bc + ab’c’

Canonical POS Form Using Truth Table

Designation	a	b	c	a’b’c + bc’ + ac’
0	0	0	0	1.1.0+0.1+0.1=0
1	0	0	1	1.1.1+0.0+0.0=1
2	0	1	0	0.0.0+1.1+0.1=1
3	0	1	1	1.0.1+1.0+0.0=0
4	1	0	0	0.1.0+0.1+1.1=1
5	1	0	1	0.1.0+0.0+1.0=0
6	1	1	0	0.0.1+1.1+1.1=1
7	1	1	1	0.0.1+1.0+1.0=0

$F(a,b,c)= \prod(0,3,5,7)= (a+b+c)\bullet(a+b'+c')\bullet(a'+b'+c')\bullet(a'+b+c')$

dec	A(8)	B(4)	C(2)	D(1)	Ab+cd
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	0	1	

Summary