2025(Backlog)

Time : 3 hours

Full Marks : 75

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer the following questions : 1×10 = 10

   (a) Define feasible solution.

   (b) Define slack variables.

   (c) Define a convex polyhedron.

   (d) Give an example of a non-convex set.

(e) What do you mean by degenerate basic feasible solution ?

(f) Write the dual of the following primal problem :

Maximize $Z_p = 3x_1 + 5x_2$

Subject to $x_1 \leq 4$

$x_2 \leq 6$

$3x_1 + 2x_2 \leq 8$

$x_1, x_2 \geq 0$

(g) Write the necessary and sufficient condition for the existence of a feasible solution to the transportation problem.

(h) Write the number of basic variables in m × n transportation problem.

(i) Write any two methods of solving transportation problem.

(j) Define assignment problem.

2. Prove that the intersection of two convex sets is also a convex set. 5

Group – B

Answer any **four** questions of the following :

3. (a) Show that the following system of linear equations has degenerate solution : 7

$2x_1 + x_2 - x_3 = 2 ; 3x_1 + 2x_2 + x_3 = 3.$

(b) Show that the set $S = \{(x_1, x_2) : 3x_1^2 + 2x_2^2 \leq 6\}$ is a convex set. 8

4. (a) Using Graphical method, solve the LPP : 7

Maximize $Z = 5x_1 + 7x_2$

Subject to $x_1 + x_2 \leq 4$

$3x_1 + 8x_2 \leq 24$

$10x_1 + 7x_2 \leq 35$

$x_1, x_2 \geq 0.$

(b) Solve the following LPP : 8

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints

$2x_1 + 3x_2 \leq 8$

$2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$x_1, x_2, x_3 \geq 0.$

5. (a) Use two-phase method to solve the following LP problem : 7

Min $Z = x_1 + x_2$

Subject to the constraints

$2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0.$

(b) Use Big-M method to solve the LPP : 8

Maximize $Z = 5x_1 - 2x_2 + 3x_3$

Subject to $2x_1 + 2x_2 - x_3 \geq 2$

$3x_1 - 4x_2 \leq 3$

$x_2 + 3x_3 \leq 5$

$x_1, x_2, x_3 \geq 0.$

6. (a) Prove that the dual of the dual of a given primal linear programming problem is a primal problem. 7

(b) Use duality to solve the following LPP : 8

Maximize $Z = 2x_1 + x_2$

Subject to $x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

$x_1, x_2 \geq 0.$

7. (a) Solve the following transportation problem by the North-West corner rule : 7

		Destinations				
		D_1	D_2	D_3	D_4	Available
Origins	O_1	19	30	50	10	7
	O_2	70	30	40	60	9
	O_3	40	8	70	20	18
Requirements		5	8	7	14	34

(b) Find the optimal solution of the following transportation problem : 8

		To			
		1	2	3	4	Supply
	1	21	16	25	13	11
From	2	17	18	14	23	13
	3	32	27	18	41	19
	Demand	6	10	12	15	43

8. (a) A company has 4 machines on which to do 3 jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table : 7

		Machine			
		A	B	C	D
	1	18	24	28	32
Job	2	8	13	17	19
	3	10	15	19	32

What are the job assignments which will minimize the cost ? Calculate the minimum cost.

(b) Solve the minimal assignment problem whose effectiveness matrix is : 8

	I	II	III	IV
A	2	3	4	5
B	4	5	6	7
C	7	8	9	8
D	3	5	8	4