2024
(Session : 2022-26)

Time : 3 hours

Full Marks : 75

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer the following questions : 1×5 = 5

   (a) Write the Bernoulli's equation.

   (b) Write the condition for the ordinary differential equation Mdx + Ndy = 0 to be exact.

(c) Write the integrating factor of the equation

$$\frac{dy}{dx} + Py = Q.$$

(d) What is the order and degree of the differential equation ?

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} - \frac{d^2y}{dx^2} = 0$$

(e) Write the Leibnitz equation.

2. Solve $\frac{d^3y}{dx^3} - 8y = 0.$ 5

3. Solve $\log\left(\frac{dy}{dx}\right) = ax + by.$ 5

Group – B

Answer any **four** questions of the following :

4. (a) Solve $\cos(x + y)dy = dx.$ 7

(b) Solve $\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}.$ 8

5. (a) Solve $(2x + y - 3)dy = (x + 2y - 3)\,dx.$ 7

(b) Write the statement of Picard's theorem for the existence and uniqueness of the solutions of the first order differential equations. Using Picard's method, solve the following initial value problem :

$$\frac{dy}{dx} = xy + 1 ; y(0) = 0.$$ 8

6. (a) Solve $x\frac{dy}{dx} + y = y^2 \log x.$ 7

(b) Solve $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0.$ 8

7. (a) Find the complete solution of $\frac{d^2y}{dx^2} + 9y = x^2.$ 7

(b) Write the Clairaut's form of differential equation. Solve $p(x^2 - a^2) - 2pxy + y^2 - b^2 = 0.$ 8

8. (a) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + a^2y = \sec ax.$ 7

(b) Solve by the method of undetermined solutions $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=x^3+x$. 8

9. (a) Prove that system of parabolas $y^2 = 4a(x + a)$ is self orthogonal. 7

(b) Define Wronskian of two functions. Check whether the following functions are independent or not : 8

$e^x\cos x$, $e^x\sin x$.