FYU-ESUE(VI) — Math (MJ.– 12)

2025

(Session : 2022-26)

Time : 3 hours

Full Marks : 75

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer **all** questions : $1 \times 5 = 5$

   (a) Define a metric space.

   (b) Define an open set in a metric space.

   (c) Give an example of a set in metric space which is both open and closed.

   (d) Define neighbourhood of a point.

(e) Find the derived set of the following subset of R :

$A =]0, 1[, B = [0, 1[$

2. Let (X, d) be a metric space and x, y, z be any three points of X then prove that : 5

$d(x, y) \geq |d(x, z) - d(z, y)|$

3. Prove that Int. (A) is an open set. 5

Group – B

Answer any **four** questions of the following :

4. (a) Let (E, f) be a metric space then prove that (E, d) is a metric space where, d is defined by : 8

$$d(x, y) = \frac{f(x,y)}{1+f(x,y)} \; \forall \, x, y \text{ of } E.$$

(b) Prove that an open sphere in a metric space (E, d) is an open set. 7

5. (a) Let F be any subset of a metric space E then prove that the derived set F' is a closed set. 8

(b) Prove that a subset F of a metric space is closed if and only if $F = \overline{F}$. 7

6. (a) Prove that every convergent sequence (x_u) of points of a metric space (E, d) is a Cauchy sequence. 8

(b) Let (X, d_1) and (Y, d_2) be metric spaces and f a mapping of X into Y then prove that f is continuous at $C \in X$ iff for every sequence (x_u) iu X $x_n \to C \Rightarrow f(x_u) \to f(C)$. 7

7. State and prove Cantor's intersection theorem. 15

8. (a) Prove that every closed subset of a compact metric space is compact. 8

(b) Prove that continuous image of a compact metric space is compact. 7

9. (a) Let A be a connected subset of a metric space X and let B be a subset of X such that $A \subseteq B \subseteq \overline{A}$ then prove that B is also connected. 8

(b) Prove that the union of two connected sets, having non-empty intersection is connected. 7