2021-24

Time : 3 hours

Full Marks : 100

Pass Marks : 40

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer the following questions : 1×10 = 10

   (a) Define Bounded Sequence.

   (b) Define monotonic Sequence.

   (c) Define Cauchy Sequence.

   (d) Define divergent series.

   (e) Define identity element of a group.

   (f) Define coset of a subgroup.

(g) Define linear differential equation.

(h) Define singular solution.

(i) Define total differential equation.

(j) What do you mean by binary operations ?

2. Prove that the series $1 + 4 + 7 + \ldots\ldots\ldots$ is divergent. 5

3. If H is any subgroup of a group G, then prove that $HH = H$. 5

Group – B

Answer any **four** questions of the following :

4. (a) State and prove Pringsheim's Theorem. 10

(b) Test the convergence of the series whose general term is $\left(1-\frac{1}{n}\right)^{n^2}$ 10

5. (a) Prove that the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}$ is convergent. 10

(b) Determine the convergency of the series

$$\frac{1}{4}+\frac{1.3}{4.7}+\frac{1.3.5}{4.7.10}+\frac{1.3.5.7}{4.7.10.13}+\ldots\ldots$$ 10

6. (a) Show that the set $G = \{a+b\sqrt{2} : a, b \in Q\}$ is a group with respect to addition. 10

(b) If H_1 and H_2 are any two subgroups of a group G, then prove that $H_1 \cap H_2$ is also a subgroup of G. 10

7. (a) Prove that any two right (left) cosets of a subgroup are either disjoint or identical. 10

(b) If a finite group of order n contains an element of order n, them prove that the group must be cyclic. 10

8. (a) Solve the differential equation $(y+1)\,p - xp^2 + 2 = 0$. 10

(b) Prove that the system of Parabolas $y^2 = 4a(x + a)$ is self orthogonal. 10

9. (a) Solve the differential equation :

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{zx}$$ 10

(b) Solve :

$$3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0.$$ 10