 FYU-ESUE(III) — Math
(MN – 1B)

2024

(Session : 2022-26)

Time : 3 hours

Full Marks : 75

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer the following questions : $1 \times 10 = 10$

   (a) Define anti-symmetric relation.

   (b) Define Denumerable set.

   (c) Define Binary operation on a set.

   (d) Define commutative group.

   (e) Define coset.

   (f) Define cyclic group.

   (g) Define normalizer of an element of a group.

(h) Define the centre of a group.

(i) Define orthogonal matrix.

(j) Define rank of a matrix.

2. Prove that the inverse of each element of a group is unique. 5

Group – B

Answer any **four** questions of the following :

3. (a) Let p be the relation on the set R of real number given by apb iff $|a-b| \leq \frac{1}{2}$ then prove that p is not an equivalence relation. 7

   (b) Prove that N × N is countable. 8

4. (a) Show that the set $G = \{1, w, w^2\}$, where w is an imaginary cube root of unity is a group with respect to multiplication. 7

   (b) If H_1 and H_2 are two subgroups of a group G, then show that $H_1 \cap H_2$ is also a subgroup of G. 8

5. (a) Prove that any two right(left) cosets of a subgroup are either disjoint or identical. 7

   (b) State and prove Lagrange's theorem. 8

6. (a) Prove that every group of prime order is cyclic. 7

   (b) To show that the multiplication of two permutations is not in general commutative. 8

7. (a) Find the rank of the matrix $\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$. 7

   (b) To solve the equations : 8

   $x + y + z = 9$

   $2x + 5y + 7z = 52$

   $2x + y - z = 0$

8. (a) Find the characteristic roots of the matrix

   $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. 7

   (b) Determine the eigenvectors of the matrix

   $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. 8

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