

2021-24*Time : 3 hours**Full Marks : 100*

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A**(Compulsory)**

1. Answer all the following questions : $1 \times 10 = 10$
 - (a) Define analytic function.
 - (b) Define harmonic function.
 - (c) Define continuity of a function.
 - (d) Define Indexed sets.
 - (e) Define countable set.

- (f) Define partial ordered set.
- (g) Define matrix.
- (h) Define row matrix.
- (i) Define unit matrix.
- (j) Define Orthogonal matrix.
2. Show that the function $u = x^3 - 3xy^2$ is harmonic and find the corresponding analytic function.

5

3. If $A = \begin{vmatrix} 2 & 3 \\ -1 & 5 \end{vmatrix}$, $B = \begin{vmatrix} 3 & 5 \\ 1 & -2 \end{vmatrix}$, $C = \begin{vmatrix} x+y & 8 \\ 0 & x-y \end{vmatrix}$,

and $A + B = C$, find the values of x and y .

5

Group – B

Answer any four questions of the following :

4. (a) State and Prove necessary condition for $f(z)$ to be analytic.
- (b) Find the analytic function $f(z) = u + iv$ of which the real part is $u = e^x (x \cos y - y \sin y)$.

10

10

5. (a) Show that an analytic function with constant modulus is constant. 10

(b) If $u = (x - 1)^3 - 3xy^2 + 3y^2$, determine v so that $u + iv$ is a regular function of $x + iy$. 10

6. (a) Prove that the set R of real number is uncountable. 10

(b) Prove that : 10

(i) $A - (B \cup C) = (A - B) \cap (A - C)$

(ii) $A' - B' = B - A$

7. (a) State and Prove Generalised De Morgan's Laws. 10

(b) If R is an equivalence relation on a set X , then show that R^{-1} is also an equivalence relation. 10

8. (a) If A be any matrix, then Prove that AA' and $A'A$ are both symmetric matrices. 10

(b) If $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix}$, find AA' and $A'A$. 10

9. (a) Find the inverse of the matrix.

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

10

(b) Show that the equations

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2$$

$$x - y + z = -1$$

are consistent and solve them.

10

