FYU-ESUE(VI) — Math (MJ – 13)

# 2025

## (Session : 2022-26)

*Time : 3 hours*

*Full Marks : 75*

*Candidates are required to give their answers in their own words as far as practicable.*

*The figures in the margin indicate full marks.*

*Answer from both the Groups as directed.*

### Group – A

### (Compulsory)

1. Answer the following questions : $1 \times 5 = 5$

   (a) Define a field.

   (b) Define a ring homomorphism.

   (c) Define integral domain.

   (d) Define principal ideal ring.

   (e) Define polynomial ring.

2. (a) Prove that the intersection of two subrings is a subring. 5

(b) The necessary and sufficient condition that the non-zero element a in the Euclidean ring R is a unit is that d (a) = d (1). 5

## Group – B

### (Descriptive Type Questions)

Answer any **four** questions of the following :

15×4 = 60

3. Let R be a commutative ring and S an ideal of R. Then prove that the ring of residue classes R / S is an integral domain if and only if S is a prime ideal.

4. Prove that a commutative ring R with identity is a field if and only if it has no proper ideals.

5. State and prove fundamental theorem an Homomorphism of rings.

6. An ideal M of a commutative ring with unity is maximal ideal if and only if R / M is a field.



7. Prove that :

(a) If R is a commutative then R [x] is commutative.

(b) If R has no proper zero divisions then R [x] also has no proper zero divisions.

8. Prove that a polynomial domain F [x] over a field F is a principal ideal ring.