

2024-28

Full Marks : 75

Time : 3 Hours

Candidates are required to give their answer in their own words as far as practicable. Their figures in the margin indicate full marks.

Answer from **both** the Sections as directed.

Section - A
(Compulsory)

1. Answer the following questions : 1×5=5

(a) Define Plane.

(b) State Leibnitz's Theorem.

(c) Define curvature.

(d) When n is even, then write the value of

$$\int_0^{\pi/2} \cos^n x \, dx$$

(e) Define quadrature.

2. State and prove the Taylor's theorem on the expansion of $f(x+h)$. 5

3. Find the angle between the line

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

and the plane $ax+by+cz+d=0$ 5

Section - B

Answer any four questions : 15×4=60

4. (a) Find the length and the equation of the shortest distance between two skew lines : 8

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} \text{ and } \frac{x-\alpha'}{l'} = \frac{y-\beta'}{m'} = \frac{z-\gamma'}{n'}$$

(b) Find the equation of the sphere through the points $(0, 0, 0)$, $(0, 1, -1)$, $(-1, 2, 0)$, $(1, 2, 3)$. 7

5. (a) State and prove Maclaurin's Theorem to expand $f(x)$ 8

(b) Expand $\cos x$ by Maclaurin's Theorem. 7

6. (a) If $y = e^{ax} \sin bx$ then find y_n . 8

(b) If $x = \sin(\log y)$. Prove that $(1-x^2)y_2 - xy_1 = y$. 7

7. (a) Find the radius of curvature for the cartesian curve $y = f(x)$. 8

(b) Find the perimeter of the circle $x^2+y^2=a^2$ 7

8. (a) Evaluate $\int_0^{\pi/2} \log \sin x \, dx$ 8

(b) Obtain a reduction formula for $\int \tan^n x \, dx$ 7

9. (a) Find the surface of the solid generated by the revolution of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis. 8

(b) Find the real asymptotes of the curve $x^3 + y^3 = 3axy$. 7
