UESE(III) — Math (CC – 5)

2024(Backlog)

Time : 3 hours

Full Marks : 75

Pass Marks : 30

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer the following questions : $1 \times 10 = 10$

   (a) Define limit of a function at a point.

   (b) Write the left hand and right hand derivatives of a function at a point.

   (c) Define uniform continuous function.

   (d) State Lagrange's Mean value theorem.

(e) Define Partition of an interval.

(f) What is upper Riemann Sum and lower Riemann Sum of a function.

(g) Write the statement of Darboux theorem.

(h) Write the fundamental theorem of integral calculus.

(i) Define upper Riemann integral of f over [a, b].

(j) What is primitive of a function ?

2. Show that the function f(x) = sin x is continuous for every real value of x. 5

Group – B

Answer any **four** questions of the following :

3. If f(x) is continuous in the closed interval [a, b] then prove that :

(a) f(x) is bounded in [a, b] 8

(b) f(x) attains its bounds atleast once in [a, b] 7

4. (a) Prove that a function f(x) = |x| is continuous at a point x = 0 but it is not differentiable at that point. 7

(b) Examine the continuity and differentiability of the function f(x) if 8

$$f(x) = \begin{cases} x \sin(1/x), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

5. (a) State and prove Rolle's theorem. 8

(b) If $f(x) = x^2$, $g(x) = x^3$ for all x. Show that there exist a number x in] – 1, 1 [for which $\frac{f(1) - f(-1)}{g(1) - g(-1)} = \frac{f'(x)}{g'(x)}$. 7

OR

Prove that if a function f is continuous on a closed interval [a, b] then it is uniformly continuous on [a, b].

6. (a) Show that a constant function is R-integrable. 7

(b) Prove that every continuous function is R-integrable. 8

7. (a) If f ∈ R [a, b] and m and M be the bounds of f on [a, b] then prove that :

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a).$$ 8

(b) Prove that the lower R-integral cannot exceed the upper R-integral. 7

8. (a) If $f \in R[a, b]$, $g \in R[a, b]$ then $fg \in R[a, b]$. 8

(b) If $f \in R[a, b]$ and $K \in R$ then $Kf \in R[a, b]$ and $\int_a^b Kf(x)\,dx = K\int_a^b f(x)\,dx$. 7