

2022*Time : 3 hours**Full Marks : 100*

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A**(Compulsory)**

1. Answer the following questions : $1 \times 10 = 10$

(a) Evaluate $\int \sin^3 x dx$.

~~$\int \sin^2 x \sin x dx$~~

(b) Find $\int \frac{dx}{\sqrt{x^2 - a^2}}$.

\cos

(c) Find $\int e^{ax+b} dx$.

(d) Find $\int \cos x e^{\sin x} dx$.

✓ (e) Evaluate $\int_0^1 \frac{dx}{e^x + e^{-x}}$.

(f) If $\vec{r}_1 = t^2\vec{i} - t\vec{j} + (2t + 1)\vec{k}$, $\vec{r}_2 = (2t - 3)\vec{i} + \vec{j} - t\vec{k}$, find $\frac{d}{dt} (\vec{r}_1 \cdot \vec{r}_2)$, where $t = 1$.

(g) Write a necessary and sufficient condition for the three non-parallel, non zero vectors \vec{a} , \vec{b} , \vec{c} to be coplanar.

(h) Write the value of $\text{Div}(\phi \vec{a})$.

(i) Simplify $(\cos \frac{\pi}{10} - i \sin \frac{\pi}{10})^{-10}$.

(j) Write the statement of De-Moivre's Theorem.

2. Integrate $\int \frac{dx}{2 \sin x + 3 \cos x}$ 5

✓ 3. Integrate $\int \frac{x^2 dx}{(x+1)(x+2)^2}$ 5

Group – B

Answer any four questions of the following :

4. (a) $\int \frac{dx}{x^3 + 1}$ 10

(b) Integrate $\int \frac{\sin x}{\sqrt{1 + \sin x}}$ 10

✓ 5. (a) Evaluate $\int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}}$ 10

✓ (b) Evaluate $\int_0^{\pi/2} \frac{\sin 2\theta d\theta}{\sin^4 \theta + \cos^4 \theta}$ 10

6. (a) Find the length of an arc of the parabola $y^2 = 4ax$ measured from the vertex to one extremity of the latus rectum. 10

(b) Evaluate $\int_0^{\pi/4} \tan^5 x dx$ using reduction formula. 10

7. Find the volume and surface area of a sphere of radius a . 20

8. (a) A particle moves along the curve $x = a \cos t$,
 $y = a \sin t$ and $z = bt$. Find the velocity and
acceleration at $t = 0$, and $t = \frac{\pi}{2}$. 10

(b) Prove that $\text{Div} (\vec{a} \times \vec{b}) = \vec{b} \cdot (\text{curl } \vec{a})$
 $-\vec{a} \cdot (\text{curl } \vec{b})$. 10

9. (a) Find $\text{div } \vec{V}$ and $\text{curl } \vec{V}$, where $\vec{V} = \nabla(x^3 + y^3$
 $+ z^3 - 3xyz)$. 10

(b) Apply De-Moivre's Theorem to solve the
equation $x^7 + x^4 + x^3 + 1 = 0$. 10

