 FYU-ESUE(III) — Math
(MJ – 5)

2024

(Session : 2022-26)

Time : 3 hours

Full Marks : 75

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer the following questions : $1 \times 5 = 5$

   (a) Define Groups.

   (b) Define Group Homomorphism.

   (c) Define Cyclic group.

   (d) Define characteristic vectors.

   (e) Define Normal subgroups.

2. Prove that $(ab)^{-1} = b^{-1} a^{-1}$, where $a, b \in G$. 5

3. Prove that the intersection of any two normal subgroups of a group is a normal subgroup. 5

Group – B

Answer any **four** questions of the following :

4. (a) If H_1 and H_2 are two subgroups of a group G, then prove that $H_1 \cap H_2$ is also a subgroup of G. 8

   (b) If a, b, c are any element of G, then prove that $ab = ac \Rightarrow b = c$ and $ba = ca \Rightarrow b = c$. 7

5. (a) State and prove Lagrange's theorem. 8

   (b) Prove that the normalizer N(a) of $a \in G$ is a subgroup of G. 7

6. (a) Prove that every finite group G is isomorphic to a permutation group. 8

   (b) Prove that number of generators of a finite cyclic group of order n is $\phi(n)$, where $\phi(n)$ is the Euler's ϕ function. 7

7. (a) State and prove Cauchy's theorem for abelian groups. 8

   (b) If f is a homomorphism of a group G into a group G' with Kernal K, then prove that K is a normal subgroup of G. 7

8. (a) State and prove First theorem of Isomorphism. 8

   (b) Prove that every homomorphic image of a group G is isomorphic to some quotient group of G. 7

9. (a) State and prove the Cayley-Hamilton theorem. 8

   (b) Determine the eigenvectors of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$ 7