NAAC ACCREDITED "B++" (CGPA 2.89)

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Course :Computer System Architecture

Class : Sem-1

Lesson :Number System Contd..

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Octal Numbering System

- Base: 8
- Digits: 0, 1, 2, 3, 4, 5, 6, 7

- Octal number: 357_{8}

$$= (3 \times 8^2) + (5 \times 8^1) + (7 \times 8^0)$$

- To convert to base 10, beginning with the **rightmost** digit, multiply each **n**th digit by $8^{(n-1)}$, and add all of the results together.

Octal to Decimal Conversion

- Example 1: 357_8

positional powers of 8: 8^2 8^1 8^0
decimal positional value: 64 8 1

Octal number: 3 5 7

$$(3 \times 64) + (5 \times 8) + (7 \times 1)$$

$$= \quad 192 + 40 + 7 = 239_{10}$$

Octal to Decimal Conversion

- Example 2: 1246_8

positional powers of 8:	8^3	8^2	8^1	8^0
decimal positional value:	512	64	8	1
Octal number:	1	2	4	6

$(1 \times 512) + (2 \times 64) + (4 \times 8) + (6 \times 1)$

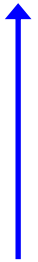
$= 512 + 128 + 32 + 6 = 678_{10}$

Decimal to Octal Conversion

Using the **Division** Method:

Example 1: $\mathbf{214_{10}}$ = $\mathbf{326_{8}}$

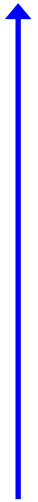
	Rem:
8) 214	
8) 26	6
8) 3	2
0	3



Decimal to Octal Conversion

Example 2: $4330_{10} = 10352_8$

	Rem:
8) 4330	
8) 541	2
8) 67	5
8) 8	3
8) 1	0
0	1



Decimal to Octal Conversion

*The **Subtraction** Method:*

- Subtract out multiples of the largest power of 8 possible (without going below zero) each time until you reach 0.
  - Place the multiple value in each position where you COULD subtract the value.
  - Place a 0 in each position that you could NOT subtract out the value without going below zero.

Decimal to Octal Conversion

Example 1: 315_{10}

315
- 256 (4 x 64)
59
- 56 (7 x 8)
3
- 3 (3 x 1)
0

8^2	8^1	8^0
64	8	1
4	7	3

Answer: $315_{10} = 473_8$

Decimal to Octal Conversion

Example 2: 2018_{10}

2018
-1536 (3 x 512)
482
- 448 (7 x 64)
34
- 32 (4 x 8)
2
- 2 (2 x 1)
0

8^4	8^3	8^2	8^1	8^0
4096	512	64	8	1
	3	7	4	2

Answer: $2018_{10} = \mathbf{3742_8}$

Hexadecimal (Hex) Numbering System

- Base: 16
- Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

- Hexadecimal number: $1F4_{16}$

$$= (1 \times 16^2) + (F \times 16^1) + (4 \times 16^0)$$

Hexadecimal (Hex)
Extra Digits

Decimal Value	Hexadecimal Digit
10	**A**
11	**B**
12	**C**
13	**D**
14	**E**
15	**F**

Hex to Decimal Conversion

- To convert to base 10:
  - Begin with the rightmost digit
  - Multiply each **n**th digit by $16^{(n-1)}$
  - Add all of the results together

Hex to Decimal Conversion

- Example 1: $1F4_{16}$

positional powers of 16: 16^3 16^2 16^1 16^0
decimal positional value: 4096 256 16 1

Hexadecimal number: 1 F 4

(1 x 256) + (F x 16) + (4 x 1)
= (1 x 256) + (15 x 16) + (4 x 1)

= 256 + 240 + 4 = 500_{10}

Hex to Decimal Conversion

- Example 2: $25AC_{16}$

positional powers of 16: 16^3 16^2 16^1 16^0
decimal positional value: 4096 256 16 1

Hexadecimal number: 2 5 A C

(2 x 4096) + (5 x 256) + (A x 16) + (C x 1)
= (2 x 4096) + (5 x 256) + (10 x 16) + (12 x 1)

= 8192 + 1280 + 160 + 12 = 9644_{10}

Decimal to Hex Conversion

The **Division** Method:

1) Start with your number (call it N) in base 10
2) Divide N by 16 and record the remainder
3) If (quotient = 0) then stop

   else make the quotient your new N, and go back to step 2

The **remainders** comprise your answer, starting with the last remainder as your first (leftmost) digit.

In other words, divide the decimal number by 16 until you reach zero, and then collect the remainders in reverse.

Decimal to Hex Conversion

Using The **Division** Method:

Example 1: $\mathbf{126_{10}} = \mathbf{7E_{16}}$

16) 126	Rem:	
16) 7	14=E	
0	7	

Decimal to Hex Conversion

Example 2: $\mathbf{603_{10}}$ = $\mathbf{25B_{16}}$

		Rem:
16) 603		
16) 37		11=B
16) 2		5
0	2	

Decimal to Hex Conversion

*The **Subtraction** Method:*

- Subtract out multiples of the largest power of 16 possible (without going below zero) each time until you reach 0.
  - Place the multiple value in each position where you COULD to subtract the value.
  - Place a 0 in each position that you could NOT subtract out the value without going below zero.

Decimal to Hex Conversion

Example 1: 810_{10}

810	
- 768	(3 x 256)
42	
- 32	(2 x 16)
10	
- 10	(10 x 1)
0	

16^2	16^1	16^0
256	16	1
3	2	A

Answer: $810_{10} = 32A_{16}$

Decimal to Hex Conversion

Example 2: 156_{10}

156

- 144 (9 x 16)

12

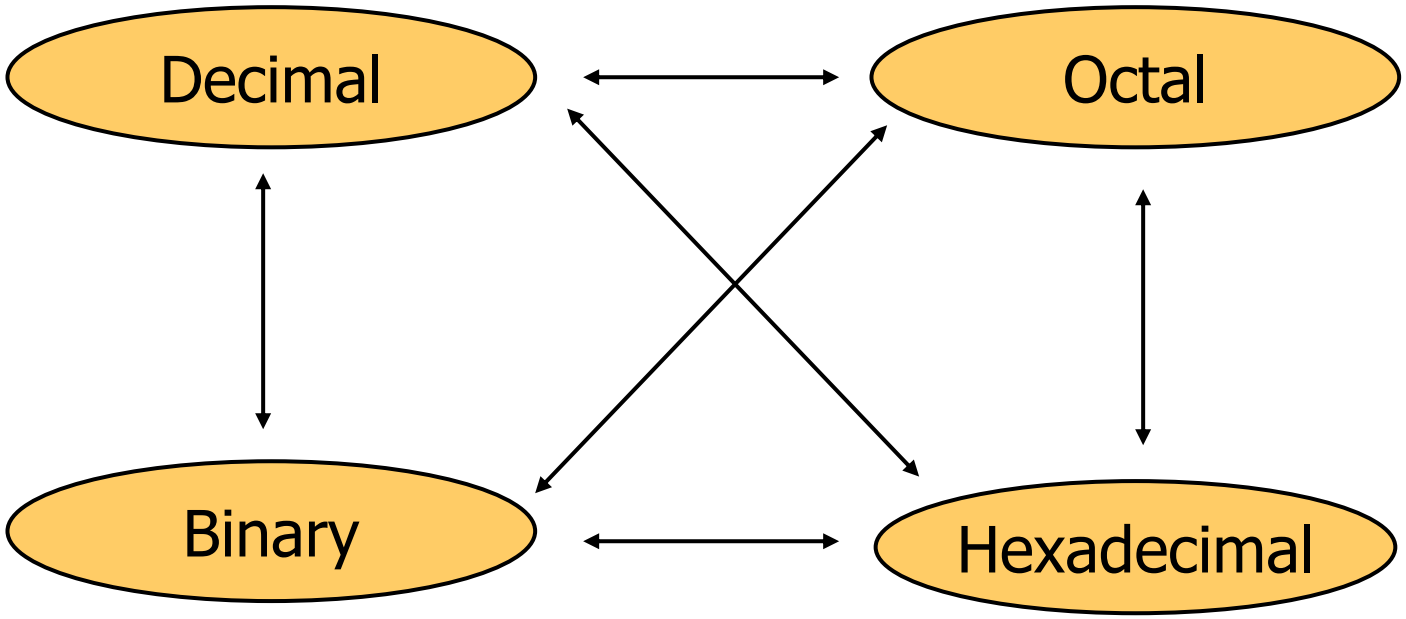
- 12 (12 x 1)

0

16^2	16^1	16^0
256	16	1
	9	C

Answer: $156_{10} = 9C_{16}$

Conversion Among Bases



Binary to Octal Conversion

The maximum value represented in 3 bit is:

$2^3 - 1 = 7$

So using 3 bits we can represent values from 0 to 7 which are the digits of the Octal numbering system.

Thus, three binary digits can be converted to one octal digit.

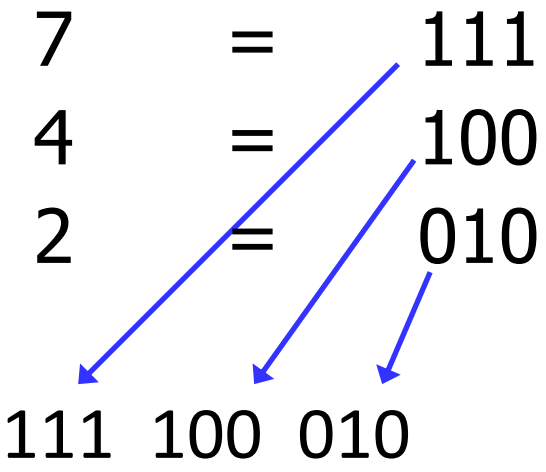
Binary to Octal Conversion

Three-bit Group	Decimal Digit	Octal Digit
000	0	**0**
001	1	**1**
010	2	**2**
011	3	**3**
100	4	**4**
101	5	**5**
110	6	**6**
111	7	**7**

Octal to Binary Conversion

Ex : Convert 742_8 to binary

Convert each octal digit to 3 bits:



$742_8 = 111100010_2$

Binary to Octal Conversion

Ex : Convert 10100110_2 to octal

Starting at the right end, split into groups of 3:

10 100 110 →

110 = 6

100 = 4

010 = 2 (pad empty digits with 0)

$10100110_2 = 246_8$

Binary to Hex Conversion

The maximum value represented in 4 bit is:

$2^4 - 1 = 15$

So using 4 bits we can represent values from 0 to 15 which are the digits of the Hexadecimal numbering system.

Thus, four binary digits can be converted to one hexadecimal digit.

Binary to Hex Conversion

Four-bit Group	Decimal Digit	Hexadecimal Digit
0000	0	**0**
0001	1	**1**
0010	2	**2**
0011	3	**3**
0100	4	**4**
0101	5	**5**
0110	6	**6**
0111	7	**7**
1000	8	**8**
1001	9	**9**
1010	10	**A**
1011	11	**B**
1100	12	**C**
1101	13	**D**
1110	14	**E**
1111	15	**F**

Binary to Hex Conversion

Ex : Convert 110100110_2 to hex

Starting at the right end, split into groups of 4:

1 1010 0110 →

0110 = 6

1010 = A

0001 = 1 (pad empty digits with 0)

$110100110_2 = 1A6_{16}$

Hex to Binary Conversion

Ex : Convert $3D9_{16}$ to binary

Convert each hex digit to 4 bits:

3 = 0011

D = 1101

9 = 1001

0011 1101 1001 →

$3D9_{16}$ = 1111011001_2 (can remove leading zeros)

Conversion between Binary and Hex - Try It Yourself

- Convert the following numbers:
  - 1010111101_2 to Hex
  - $82F_{16}$ to Binary
- (Answers on NEXT slide)

Answers

- 1010111101_2 → 10 1011 1101
  = $2BD_{16}$

- $82F_{16}$ = 0100 0010 1111
  → 100001011111_2

Octal to Hex Conversion

- To convert between the Octal and Hexadecimal numbering systems
  - Convert from one system to binary first
  - Then convert from binary to the new numbering system

Hex to Octal Conversion

Ex : Convert $E8A_{16}$ to octal

First convert the hex to binary:

$1110\ 1000\ 1010_2$

111 010 001 010 and re-group by 3 bits (starting on the right)

Then convert the binary to octal:

7 2 1 2

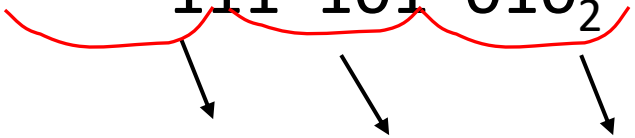
So $E8A_{16} = 7212_8$

Octal to Hex Conversion

Ex : Convert 752_8 to hex

First convert the octal to binary:

$111\ 101\ 010_2$



re-group by 4 bits

0001 1110 1010 (add leading zeros)

Then convert the binary to hex:

1 E A

So $752_8 = 1EA_{16}$

Thank You