

2025(Backlog)

Time : 3 hours

Full Marks : 75

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Use of scientific calculator is allowed.

Group – A

(Compulsory)

1. Choose the correct answer of the following to fill in the bank : 1×10 = 10

(a) $\left\{\frac{\Delta^2}{E}\right\}x^3 =$ _____.

(i) 1 (ii) 2x

(iii) 4x (iv) 6x

(b) The first term of the series whose second and subsequent terms are 8, 3, 0, – 1, 0 is _____.

(i) 14 (ii) 15

(iii) 16 (iv) 17

(c) $\Delta \tan^{-1}\left(\frac{x-1}{x}\right) =$ _____.

(i) $\tan^{-1}\frac{1}{x}$

(ii) $\tan^{-1}\frac{1}{2x}$

(iii) $\tan^{-1}\frac{1}{x^2}$

(iv) $\tan^{-1}\frac{1}{2x^2}$

(d) If $f(x) = \frac{1}{x^2}$, then the divided difference $\delta(a, b) =$ _____.

(i) $\frac{a+b}{ab}$

(ii) $\frac{a-b}{ab}$

(iii) $\frac{a+b}{a^2b^2}$

(iv) $-\frac{a+b}{a^2b^2}$

(e) $\Delta \log f(x) =$ _____.

(i) $\log\left(\frac{\nabla f(x)}{f(x)}\right)$

(ii) $\log\left(\frac{\Delta f(x)}{f(x)}\right)$

(iii) $\log\left(\frac{Ef(x)}{f(x)}\right)$

(iv) $\log\left(\frac{f(x)}{Ef(x)}\right)$

(f) The curve passing through the points (0, 4), (1, 3), (4, 24) and (5, 39) is _____.

(i) $2x^2 - 3x + 4$

(ii) $x^2 - 3x + 4$

(iii) $2x^2 + 3x - 4$

(iv) $x^2 + 3x - 4$

(g) If $A = \begin{bmatrix} 50 & 107 & 36 \\ 25 & 54 & 20 \\ 31 & 66 & 21 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$

$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$, then $l_{21}u_{13} + u_{23} =$ _____.

(i) 20 (ii) 31

(iii) 36 (iv) 66

(h) For the table :

x	f(x)
$x_0 = 0$	3
$x_1 = 1$	6
$x_2 = 2$	11
$x_3 = 3$	18
$x_4 = 4$	27

$\Delta^2 f(x_2) =$ _____.

(i) 2 (ii) – 2

(iii) 4 (iv) – 4

(i) Let $\frac{dy}{dx} = x + y$; $y(0) = 1$. Picard's first approximation $y_1 =$ _____.

(i) $1 + x$

(ii) $1 + x + \frac{x^2}{2}$

(iii) $1 - x$

(iv) $1 - x + \frac{x^2}{2}$

2. Find a real root of the equation $x^3 - x - 1 = 0$ by Bisection Method. 5

Group – B

Answer any **four** questions the following :

3. (a) Find a positive root of the equation $\log_{10} x - 2x + 7 = 0$ by Newton-Raphson's method. 8

(b) Solve the system of equations $2x + y + 4z = 12$, $8x - 3y + 2z = 20$, $4x + 11y - z = 33$ by Gauss's elimination method. 7

4. (a) Find the cubic polynomial y(x) such that $y(0) = 1$, $y(1) = 0$, $y(2) = 1$ and $y(3) = 10$. Hence or otherwise find y(4). 8

(b) Obtain the estimate of the missing figures in the following : 7

x	f(x)
1	1
2	8
3	?
4	64
5	?
6	216
7	343
8	512

5. (a) Given $\log_{10}654 = 2.8156$, $\log_{10}658 = 2.8182$, $\log_{10}659 = 2.8189$, $\log_{10}661 = 2.8202$. Find $\log_{10}656$. 8

(b) The population of a certain town during the last four census are given below. Estimate the population of the town in the year 1986 by Newton's forward difference interpolation formula : 7

Year	Population in lakh
1981	20
1991	27

Year	Population in lakh
2001	39
2011	52

6. (a) Find the first and second derivative of the function from the following at the point x = 1.5 : 8

x	f(x)
1.5	3.37
2	7
2.5	13.62
3	24
3.5	38.87
4	59

(b) Using divided difference, find the value of $f'(8)$, given that $f(6) = 1.556$, $f(7) = 1.690$, $f(9) = 1.908$, $f(12) = 2.158$. 7

7. (a) Derive a general quadrature formula for the integral $\int_{x_0}^{x_n} y\,dx$ where $y = f(x)$ by dividing the interval $[x_0, x_n]$ into n equal parts and hence obtain Simpson's one third rule. 8

(b) Evaluate using Simpson's one third rule $\int_1^4 \frac{dx}{2x+1}$ by taking 7 equispaced ordinates. 7

8. (a) Apply Picard's method to find the solution of the problem $\frac{dy}{dx} = y - x$; $y(0) = 2$. Show that the iterative solution approaches the exact solution $1 + x + e^x$. 8

(b) Using Picard's method, solve $\frac{dy}{dx} = 1 + xy$; $y(0) = 2$. Find $y(0.1)$ and $y(0.2)$. 7