FYU-ESUE(VI) — Math (MJ – 14)

2025

(Session : 2022-26)

Time : 3 hours

Full Marks : 75

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer the following questions : $1 \times 5 = 5$

   (a) Define Independent Events.

   (b) Define Moment Generating Function.

   (c) Define Negative Binomial Distribution.

   (d) Write characteristics of a good Estimator.

   (e) Distinguish between discrete and continuous Random Variable.

2. (a) If A and B are independent events then prove that $\bar{A}$ and $\bar{B}$ are also independent. 5

(b) Ten coins are thrown simultaneously. Find the probability of getting at least eight Heads. 5

Group – B

Answer any **four** questions of the following :

3. (a) State and prove Baye's Theorem. 8

(b) A continuous random variable X has a p. d. f. $f(x) = 3x^2, 0 \le x \le 1$. Find 'a' and 'b' such that (i) $P(x \le a) = P(x > a)$ and (ii) $P(x > b) = 0.05$. 7

4. (a) Let X be a random variable with the following probability distribution : 8

X	Pr(X = x)
–3	$\frac{1}{6}$
6	$\frac{1}{2}$
9	$\frac{1}{3}$

Evaluate $E(2x + 1)^2$.

(b) Prove that $E(x) = E[E(X/Y)]$. 7

5. (a) Two random variables X and Y have the following joint probability density function :

$f(x, y) = 2 - x - y \,;\, 0 \le x \le 1, 0 \le y \le 1 = 0$, otherwise.

Find :

(i) Marginal probability density functions of X and Y.

(ii) Conditional density functions. 8

(b) Find the moment generating function of the random variable whose moments are $\mu'_r = (r + 1)!\, 2^r$. 7

6. (a) If the independent random variables X, Y are binomially distributed respectively with n = 3, $p = \frac{1}{3}$ and n = 5, $p = \frac{1}{3}$. Write down the probability that $X + Y \ge 1$. 8

(b) The two independent random variables X_1 and X_2 have the same geometric distribution. Show that the conditional distribution of $X_1 / (X_1 + X_2 = n)$ is uniform. 7

7. (a) A random sample of 500 pineapples was taken from a large consignment and 65 were found to be bad. Show that the Standard Error of the bad ones is the

sample of this size is 0.015 and prove that the percentage of bad pineapples in the consignment almost certainly lies between 8.5 and 17.5. 8

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with continuous density. Show that $y_1 = \min(X_1, X_2, \ldots, X_n)$ is exponential with parameter $n\lambda$ if each X_i is exponential with parameter λ. 7

8. (a) If $X_1, X_2, \ldots, X_n$ are random observations on a Bernoulli Variate x taking the value 1 with probability p and the value 0 with propability (1 – p). Show that $\frac{\Sigma x_i}{n}\left(1-\frac{\Sigma x_i}{n}\right)$ is a consistent estimator of p(1 – p). 8

(b) In two large populations, there are 30 and 25 percent respectively of blue-eyed people. Is this difference likely to be hidden in the samples of 1200 and 900 respectively from the two populations. 7