2025

(Session : 2023-25)

Time : 3 hours

Full Marks : 70

Pass Marks : 28

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer the following questions : 1×5 = 5

   (a) Write down the Schmidt explict formula for solving $u_t = u_{xx}$.

   (b) What is the Crank-Nicolson formula for solving one-dimensional heat equation ?

(c) Write the name and equations of any two common elliptic partial differential equations.

(d) What do you mean by Dirichlet boundary condition ?

(e) What do you mean by stability of the difference scheme ?

2. Describe three level explict method for solving

$$\frac{\partial u}{\partial t} = \frac{\partial^2 x}{\partial x^2}.$$ 5

Group – B

Answer any **four** questions of the following :

3. (a) Discuss Von-Neumann stability analysis of the finite difference scheme. 7

(b) Show that the Schmidt method is conditionally stable. 8

4. Consider the initial boundary value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}$$

$u(r, 0) = J_0(\alpha r), 0 \le r \le 1$

$\frac{\partial u}{\partial r}(0, t) = 0, u(1, t) = 0, t > 0$

where α is a root of $J_0(\alpha) = 0$. Use the Crank Nicolson method with $h = \frac{1}{3}$ and $\lambda = \frac{1}{4}$ to integrate for one time step. 15

5. Solve the partial differential equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0 = y$, $x = 3 = y$ with $u = 0$ on the boundary and mesh length = 1. 15

6. Discuss the solution of elliptic equations by Relaxation method. 15

7. Solve the Poisson equation 15

$u_{xx} + u_{yy} = -81xy, 0 < x < 1, 0 < y < 1,$

given that $u(0, y) = 0$, $u(x, 0) = 0$

$u(1, y) = 100, u(x, 1) = 100$

and $h = \frac{1}{3}$.

8. Solve the elliptic equation 15

$u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown :